B.Tech-4

Math-IV

Set-1

MATHEMATICS-IV

Full Marks : 70

Time : 3 hours

Answer **Q. No. 1** which is compulsory and any **five** questions out of the rest seven questions

The figures in the right-hand margin indicate marks

1. Answer *all* parts of this questions : 2×10

(*a*) Explain rounding and chopping of a number and the error involved in both the cases.

(*b*) State the condition of convergence of the fixed point iteration method.

(*c*) Prove that the interpolating polynomial for a function $f(x)$ at $x_0, x_1, x_2, \ldots, x_n$ is unique.

(*d*) Evaluate $I = \int_0^1 e^{-x^2}\, dx$, by Trapezoidal rule with $n = 6$.

(*e*) State the condition on $f(x)$ for error estimation and mention the error formula in Simpson's $\frac{1}{3}$ rule of integration.

(*f*) What is a degenerate feasible solution in an LPP ?

(*g*) Prove that $1+\mu^2\sigma^2 = \left(1+\frac{\delta^2}{2}\right)^2$.

(*h*) Five coins are tossed simultaneously. Find the probability of the event A : At least one head turns up. Assume that the coins are fair.

(*i*) State the normal distribution function and plot it.

(*j*) Define random variable.

(Continued)

2. (*a*) Find a positive root of $x^4 - x = 10$ using Newton's method, do only three iterations by suitably choosing a point. 5

(*b*) Find a form $x = g(x)$ of $f(x) = x^2 - 2 = 0$, that gives convergence to the root near 1. Perform 4 iterations to find the root x starting with 1. 5

3. (*a*) Using Newton's forward formula compute the pressure of the steam at temperature 142° from the following steam table : 5

Temperature	140	150	160	170	180
Pressure	3·685	4·854	6·302	8·076	10·225

(*b*) Integrate $f(x) = \frac{1}{4}\pi x^4 \cos\frac{1}{4}\pi x$, from $x = 0$ to 2 by Simpson's $\frac{1}{3}$rd rule with $n = 5$. 5

4. (*a*) Starting from $x = 1$, $y = 1$, $z = 1$, solve the following system of linear equations by Gauss-Seidel method (do three iterations) : 5

$$5x + y + 2z = 19$$
$$x + 4y - 2z = -2$$
$$2x + 3y + 8z = 39.$$

(*b*) Apply power method starting from $[1, 1, 1]^T$ to compute the eigenvector and eigenvalue of the following matrix, do three iterations.

$$\begin{pmatrix} 3{\cdot}6 & -1{\cdot}8 & 1{\cdot}8 \\ -1{\cdot}8 & 2{\cdot}8 & -2{\cdot}6 \\ 1{\cdot}8 & -2.6 & 2{\cdot}8 \end{pmatrix}.$$ 5

5. (*a*) Solve the initial value problem $y' + 5x^4y^2 = 0, y(0) = 1, h = 0{\cdot}2$ by Eulers method. 5

(*b*) Using Runge-Kutta 4th order method find the solution of $y' = y - x, y(0) = 1{\cdot}5$ on $[0, 1]$ by choosing $h = 0{\cdot}2$. 5

6. (*a*) Solve the LPP : 5

Maximize $z = 30x_1 + 10x_2$,

subject to $-x_1 + x_2 \leq -5$,

$2x_1 + x_2 \geq 10$,

$x_2 \geq 4$, $10x_1 + 15x_2 \leq 150$.

(*b*) Using an artificial variable : 5

Minimize $f = 4x_1 + x_2 + 2x_3$

subject to $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$,

and $x_1 + x_2 = x_3 \leq 0$, $x_1 + x_2 - x_3 \leq 0$.

7. (*a*) Five screws are drawn at random from a lot of 100 screws, 10 of which are defective. Find the probability of the event that all 3 screws drawn are non defective, assuming that we draw (*i*) with replacement (*ii*) without replacement. 5

(*b*) Sketch the density $f(x) = \frac{1}{4}$ $(2 < x < 6)$ and the distribution function. Find $P(X \geq 4)$, $P(X \leq 3)$. 5

8. (*a*) Determine a 95 % confidence interval for the mean μ of a normal population with variance $\sigma^2 = 16$, using a sample of 200 with mean 74·81. 5

(*b*) Compute the correlation coefficient between *X* and *Y*. 5

$X =$	1	2	3	4	5	6	7	8	9
$Y =$	123	106	114	128	113	109	120	102	111